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Two Needles in Exponential Haystacks

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Abstract

Erdős Magic, more formally known as The Probabilistic Method, is a powerful tool for proving the existence of a combinatorial object, such as a coloring. A probability space is created for which the probability of success is positive. Hence the desired object must exist. But where is it? Here we examine instances in which the probability is exponentially small so that a standard randomized algorithm would not be in P. Nonetheless, we give two recent startling successes.

Let $\vec{v_1}, \ldots, \vec{v_n} \in R^n$ with all coordinates in $[-1, +1]$. A quarter century ago this speaker showed that there exists $\vec{x} \in R^n$ with all coordinates ± 1 so that all $|\vec{x} \cdot \vec{v_i}| \leq K\sqrt{n}$ where K is an absolute constant. (This proved and generalized a conjecture of Erdős on the discrepency of a family of n sets on n elements.) He long conjectured that no polynomial time algorithm could find the coloring. Wrong! Shachar Lovett and Raghu Meka place the problem in a geometric setting, in which the coloring $\vec{x} = (x_1, \ldots, x_n)$ is initially $\vec{0}$ and moves to the boundary of $[-1, +1]^n$ continuously. The moment is a (carefully!) modified Brownian motion.

Even longer ago, László Lovász, with the Lovász Local Lemma, showed (roughly!) that when bad events are mostly independent there is a positive probability that the random object has no bad events. In particular, an instance of k-SAT in which each clause overlaps at most $2^k/d$ other clauses is satisfiable. His proof did not yield the satisfying assignment. Robin Moser gives an amazingly simple "fix-it" randomized algorithm to find the object.

In both cases the algorithmic proofs are quite different from the original arguments.

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