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HERMITIAN EIGENVALUE PROBLEM AND ITS GENERALIZATION TO ANY SEMISIMPLE GROUP: A SURVEY

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Abstract

The classical Hermitian eigenvalue problem asks the following question: What are the possible eigenvalues of the sum $A + B$ of two $n \times n$ Hermitian matrices A and B , provided we fix the eigenvalues of A and B . A systematic study of this problem was initiated by H. Weyl (1912). By the contributions from a long list of mathematicians, notably Weyl (1912), Horn (1962), Klyachko (1998) and Knutson-Tao (1999), the problem was finally settled. The solution asserts that the eigenvalues of $A + B$ are given in terms of certain system of linear inequalities in the eigenvalues of A and B . These inequalities are given explicitly in terms of certain triples of Schubert classes in the singular cohomology of Grassmannians and the standard cup product. The Hermitian eigenvalue problem has been extended by Berenstein-Sjamaar (2000) and Kapovich-Leeb-Millson (2005) for any semisimple complex algebraic group G . Their solution is again in terms of linear inequalities obtained from certain triples of Schubert classes in the singular cohomology of the partial flag varieties G/P (P being a maximal parabolic subgroup) and the standard cup product. However, their solution is far from being optimal. In a joint work with P. Belkale, we have given an optimal solution of the problem for any G . We define a deformation of the cup product in the cohomology of G/P and use this new product to generate certain inequalities which optimally solves the problem for any G .

The talk should be accessible to general mathematical audience.

Date : Friday, December 15, 2023

Time: 13:30

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