I M B M istanbul matematiksel bilimler merkezi
istanbul center for mathematical sciences

From Large Deviations to Statistical Mechanics: What Is the Most Likely Way for an Unlikely Event To Happen?

Richard S. Ellis

University of Massachusetts Amherst

Abstract

This talk is an introduction to the theory of large deviations, which studies the asymptotic behavior of probabilities of rare events. The talk is accessible to a general mathematical audience including graduate students. The theory of large deviations has its roots in the work of Ludwig Boltzmann, the founder of statistical mechanics. In 1877 he did the first large deviation calculation in science when he showed that large deviation probabilities of the empirical vector could be expressed in terms of the relative entropy function. In this talk Boltzmann's insight is applied to prove a conditional limit theorem that addresses a basic issue arising in mathematics, statistical mechanics, and other applications. What is the most likely way for an unlikely event to happen? This question is answered in the context of n tosses of a cubic die and other random experiments involving finitely many outcomes. Let X_i denote the outcome of the i'th toss and define $S_n = X_1 + ... + X_n$. If the die were fair, then one would expect that for large n, S_n/n should be close to the theoretical mean of 3.5. Given that n is large but that S_n/n is close to a number z not equal to 3.5, the problem is to compute, in the limit n to infinity, the probability of obtaining $k = 1, 2, 3, 4, 5, 6$ on a single toss. Interestingly, this conditional limit theorem is intimately related to statistical mechanics because it gives a rigorous derivation, for a random ideal gas, of a basic construction due to Gibbs; namely, the form of the canonical ensemble from the microcanonical ensemble. A related conditional limit theorem for the distribution of X_1, X_2 illustrates the phenomenon of propagation of chaos.

Date : Monday, July 2, 2012

Time: 14:00

Place: IMBM Seminar Room, Boğaziçi University South Campus