ESF Exploratory Workshop

Interfaces of Noncommutative Geometry with the Representation Theory of Hopf Algebras and Artin Algebras

Tuesday August 07, 2012 (IMBM Seminar Room)

09:45-10:45 Gabriella Bohm (Wigner Research Centre For Physics)

Title: On the category of weak bialgebras

Abstract: Weak (Hopf) bialgebras are described as (Hopf) bimonoids in appropriate duoidal (also known as 2-monoidal) categories. This interpretation is used to define a category wba of weak bialgebras over a given field. As an application, the "free vector space" functor from the category of small categories with finitely many objects to wba is shown to possess a right adjoint, given by taking "groupoid-like" elements. This adjunction is proven to restrict to the full subcategories of groupoids and of weak Hopf algebras, respectively.

11:00-12:00 Dragos Stefan (University of Bucharest)

Title: Koszul pairs (joint work with Pascual Jara Martinez and Javier Lopez Peña).

Abstract: Koszul rings were introduced by Beilinson, Ginzburg and Sorgel as a natural generalization of Koszul algebras. In order to investigate some homological properties of this class of graded rings we introduce Koszul pairs, and then we briefly present their basic properties. The main examples of Koszul pairs are associated to certain braided bialgebras in the category of bimodules over a semisimple ring. Twisted tensor products of algebras provide other important examples. We further discuss how our results on Koszul pairs can be exploited to investigate Hochschild (co)homology of Koszul rings.

Lunch Break

13:30-14:30 Laiachi El Kaoutit Zerri (University of Granada)

Title: The algebraic groupoid structure of the universal Picard-Vessiot ring, differential operators and Jet spaces.

Abstract: In this talk we will see how the affine scheme $Spec(\mathcal{V})$ represented by the universal Picard-Vessiot ring $\mathcal{V}$ of affine line $\mathbb{A}^1_{\mathbb{C}}$ over the complex numbers, admits a structure of affine algebraic groupoid, or equivalently $\mathcal{V}$ is a commutative Hopf algebroid over the coordinate ring. This affine scheme has an open cover given by all affine algebraic groups attached to all matrix linear differential equations. The category of all differential modules is then recognized as the Cauchy category of (right) comodules over $\mathcal{V}$. This will be a specialization of a general result which we will show and deals with certain duality between commutative Hopf algebroids and cocommuative (right) Hopf algebroids with the source equal to the target. If times allows, then we will see that in the case of the affine line, this duality is strongly related to the well known duality between the differential operators and Jet spaces.

14:45-15:45 Kürşat Aker (Bilgi University)

Title: Sage for beginners.

Abstract: This is going to be the first of the two sessions Kürşat Aker will conduct. The aim of the computer algebra sessions is to demonstrate the state-of-the-art in computer algebra software and how it can be used effectively in cutting edge research to perform interactive experiments.

Wednesday August 08, 2012 (IMBM Seminar Room)

09:45-10:45 Mercedes Siles Molina (Universidad de Malaga)

Title: Structure theory for Leavitt path algebras

Abstract: There are a number of ways to associate an algebra to a graph, the most common one is the so called path algebra. Starting from this path algebra and taking suitable quotients, we arrive at deep matrices, Bergman algebras, Cohn path algebras, Leavitt path algebras and graph C*-algebras (if we add the structure of a C*-algebra).

Building on the pioneering work by Cuntz and Krieger in the 1980's, the study of graph C*-algebras took a rapid development. The investigation of Leavitt path algebras started more recently, around 2004. These can be considered as the algebraic analogue of graph C*-algebras; they also can be seen as a generalization of the algebras studied by Leavitt in a quest for algebras A without the property that all the bases of a given A-module have the same cardinality.

Many known algebras and C*-algebras can be described as the graph C*-algebra or as the Leavitt path algebra associated to a particular graph. Consequently, many structural properties can be expressed in terms of the underlying graph; examples are: simplicity, being simple and purely infinite, von Neumann regularity, stable rank, and others. This provides us with a convenient way of producing algebras and C*-algebras 'à la carte'.

There are deep connections between Leavitt path algebras, graph C*-algebras and other branches of Mathematics. We mention here the applications in K-theory and to Elliott's classification programme for certain separable nuclear C*-algebras. In our talk, we will introduce some of the ideas above, will go in deep on the structure of Leavitt path algebras and elaborate on this fascinating interplay between Analysis and Algebra.

11:00-12:00 Ken Goodearl (University of California, Santa Barbara)

Title: Prime Spectra in Algebras with Torus Actions

Abstract: When an algebra A is viewed as a noncommutative coordinate ring of an affine variety V , the primitive ideals of A are natural objects to take over the role of points, and the primitive spectrum Prim A with its Zariski topology is an appropriate "noncommutative affine variety" corresponding to V. Key problems are to

(a) Identify the points of Prim$A$.

(b) Describe the Zariski topology on Prim$A$.

(c) Relate Prim$A$ to V.

For a large class of algebras, including many quantized coordinate rings, one can reach (a) and part of (b) via a framework coming from a torus action. Specifically, suppose that A is an algebra over an infinite field k, and that there is a rational action of a k-torus $H = (k^{\times})^r r$ on A (by k-algebra automorphisms) such that A has only finitely many H-prime ideals (= prime ideals invariant under H). In this setting, there is a partition of the prime spectrum Spec$A$ into subsets $\mathrm{Spec}_J A$ indexed by the H-prime ideals J of A such that (1) $\mathrm{Spec}_J A$ is homeomorphic to the prime spectrum of the center of a localization A_J of A/J; and (2) $Z(A_J)$ is a commutative Laurent polynomial ring over a field k_J . Under some mild additional assumptions, the coefficient fields k_J coincide with the base field k, and Prim$A$ consists exactly of those prime ideals of A which are maximal within one of the sets $\mathrm{Spec}_J A$. Thus, in this case Prim$A$ is a finite disjoint union of classical (commutative) affine varieties. We will discuss the ideas involved in this framework, and survey classes of algebras to which it applies. A key example, the multiparameter quantized coordinate ring of $GL_2(k)$, will be discussed in detail.

Lunch Break

13:30-14:30 Kürşat Aker (Bilgi University)
Title: Quantum Gelfand Pairs

Abstract: Given a finite group G, a representation V is called multiplicity-free, if no irreducible representation of G shows up more than once in V. A pair of finite groups, (G, K) is said to form a Gelfand pair when the permutation representation obtained from the G-set G/K is a multiplicity-free G-representation. For instance,

> For any positive integers m and n, the pair $(S(m+n), S(m) \times S(n))$ is a Gelfand pair, where $S(k)$ stands for the symmetric group on k letters.

Woronowicz introduced the compact matrix quantum groups in 1987. Wang defined quantum symmetry groups of finite spaces as compact matrix quantum groups in the sense of Woronowicz ten years later. Let $A_s(k)$ denote the quantum symmetric group on k letters. In this talk, we will present the following result:

For any positive integers m and n, the pair $(A_s(m+n), A_s(m) \star A_s(n))$ is a Gelfand pair, where $\star$ stands for free product.

This is a joint work with Mohan Ravichandran, Sabanci University.

14:45-15:45 Birge Huisgen-Zimmermann (University of California, Santa Barbara)

Title: The finitistic dimension conjectures for finite dimensional algebras.

Abstract: I will start with a brief survey of pre-2000 results on the Finitistic Dimension Conjectures (which date back to 1960), and then discuss progress made post-2000. In particular, I will address:

- Jumps in the functions $n \mapsto findim_n\Lambda$ for $n \in \mathbb{N}$, where Λ is a finite dimensional algebra and $findim_n\Lambda$ is the supremum of the **finite** projective dimensions attained on n-generated left Λ-modules.
- The recent discovery that representation-tame algebras may fail to be "homologically tame".

I will give some concrete illustrations along the way and will conclude with an example – in my opinion the simplest – realizing a gap between the big and little finitistic dimensions.

16:15-17:15 Discussion Panel

A Panel of discussion on the current condition and possible future directions in research topics discussed so far.

Thursday August 09, 2012 (IMBM Seminar Room)

9:45-10:45 Stefan Caenepeel (Vrije Universiteit Brussels)

Title: 2- and 3-cocycles versus the bicategories of monoids and pseudomonoids

Abstract: We introduce cohomology of complexes of restricted Picard groupoids. Restricted Picard groupoids can be viewed as the appropriate categorical version of abelian groups. To such a complex, we associate a sequence of bicategories. Equivalence classes in these bicategories are abelian groups, and these are the corresponding cohomology groups. To a commutative R-bialgebroid A, we can associate a complex of restricted Picard groupoids. We provide interpretations of the associated bicategories at level 2 and 3.

Level 2: A-module corings can be organized into a bicategory, and the 2-cocycle bicategory is a subbicategory of this; in case A is a Hopf algebroid, the 0-cells are A-Galois coobjects.

Level 3: The category of commutative $R \otimes R$-rings is a bicategory. Pseudomonoids in this category with underlying 0-cell A form again a bicategory. The 3-cocycle bicategory is a subbicategory of this bicategory. This construction is related to the notion of quasibialgebra.

11:00-12:00 Alessandro Ardizzoni (Università degli Studi di Torino)

Title: Bosonization for Dual Quasi-Bialgebras and Preantipode.

Abstract: It is known that a dual quasi-bialgebra H with antipode, i.e. a dual quasi-Hopf algebra, fulfils a fundamental theorem for right dual quasi-Hopf H-bicomodules. The converse in general is not true. We show that, for a dual quasi-bialgebra H, the structure theorem is equivalent to the existence of a suitable endomorphism of H that we call a preantipode of H. We associate a dual quasi-bialgebra, named bosonization, to every dual quasi-bialgebra H and every bialgebra R in the category of Yetter-Drinfeld modules over H. The behavior of the notions of preantipode and bosonizaion is then investigated in some specific cases.

This talk is based upon the papers:

1) A. Ardizzoni, A. Pavarin, *Preantipodes for Dual Quasi-Bialgebras*, Israel J. Math., to appear.

2) A. Ardizzoni, A. Pavarin, *Bosonization for Dual Quasi-Bialgebras and Preantipode*, submitted. (arXiv:1111.4325)

Lunch Break

13:30-14:30 Claudia Menini (University of Ferrara, Ferrara, Italy)
Title: Monadic Decompositions and Classical Lie Theory

Abstract: Following [MS, AHT] we introduce the concept of monadic length for a right adjoint functor R. As a main application we show that the functor from bialgebras to vector spaces sending a bialgebra to its subspace of primitives has monadic lenght 2. The enveloping algebra of a suitable classical Lie algebra plays a fundamental role in this proof.

This is part of a joint research with A. Ardizzoni and J. Gòmez-Torrecillas

[AHT] J. Adámek, H. Herrlich, W. Tholen, Monadic decompositions. J. Pure Appl. Algebra 59 (1989), no. 2, 111-123.

[MS] J. L. MacDonald, A. Stone, The tower and regular decomposition. Cahiers Topologie Géom. Différentielle 23 (1982), no. 2, 197-213.

14:45-15:45 Emily Burgunder (Institut Mathématiques Toulouse)
Title: Finer structures on combinatorial Hopf algebras

Abstract: We introduce two combinatorial Hopf algebra encoding respectively for the hyperplane arrangement and the Stasheff polytope. We show that they can be endowed with an algebra structure finer than the associative structure namely the tridendriform algebra structure. We will show how a generalisation of the Poincaré-Birkhoff-Witt theorem to tridendriform bialgebras will help us to give a basis as free tridendriform algebras.

16:15-17:15 Kürşat Aker (Bilgi University)
Title: Sage for the initiated.

Abstract: This is going to be the second of the sessions conducted by Kürşat Aker (Bilgi University). The aim of the computer algebra sessions is to demonstrate the state-of-the-art in computer algebra software and how it can be used effectively in cutting edge research to perform interactive experiments.

Friday August 10, 2012 (IMBM Seminar Room)

9:45-10:45 Jake Goodman (University of Glasgow)

Title: Untwisting a twisted Calabi-Yau algebra

Abstract: Recent interest in Calabi-Yau algebras (algebras satisfying Poincare duality in their Hochschild (co)homology) has spread to the study of twisted Calabi Yau algebras, a weakening of the original definition in which one possibly has to twist coefficients by an algebra automorphism. One may view twisted Calabi-Yau algebras as a derived analogue of Frobenius algebras and from this point of view, Calabi-Yau algebras correspond to symmetric Frobenius algebras. In this talk, I will present a method for producing a Calabi-Yau algebra from a twisted Calabi-Yau algebra.

11:00-12:00 Zoran Skoda (University of Zagreb)

Title: Hopf torsors over corings

Abstract: A noncommutative space can often be presented by a noncommutative cover by affines, i.e. by spaces represented by noncommutative algebras. Such a cover can be presented by a coring with a distinguished group like element, as in the 1998 work of Kontsevich and Rosenberg. Given such a cover and a Hopf algebra H, I will present a generalization of cleft H-comodule algebras over a base space presented by a cover, where the total space is also given by a cover, together with an appropriate version of a coaction of H. Such a Spec(H)-torsor can be equivalently given by a new kind of Čech like cocycle data over the base. This concept is studied in a work in progress with Tomasz Brzeziński.

Lunch Break

13:30-14:30 Georgy Sharygin (ITEP)

Title: Obstructions for the deformation quantization of integrable systems

Abstract: According to a famous result of Kontsevich for every Poisson manifold there exists a unique (up to an isomorphism) deformation quantization, i.e. a non-commutative product on the algebra of functions on this manifold, such that its first order terms are determined by the Poisson structure. One can ask a rather straightforward question: suppose, we have not just a Poisson manifold, but an integrable system, i.e. Poisson manifold with a Poisson-commutative subalgebra inside it. It is always possible to choose the quantization so that the subalgebra remains commutative? It turns out, that there are certain set of cohomological obstructions, which can prevent this from being true. In my talk (based on a joint work with D.Talalaev, ITEP), I will try to explain this construction and also speculate on possible reformulation of the problem in the terms of formality of certain DGL module.

14:45-15:45 K. Worytkiewicz (Savoie University, Cedex, France)

Title: Hopf Algebras from adjunctions.

Abstract: We elaborate on a construction due to J.Ayoub which, under suitable assumptions, produces a bi-algebra H from an adjoint pair (F, U) among symmetric monoidal categories. H is universal in the sense that the left adjoint F factors through the category of comodules $\mathrm{Comod}(H)$ over H and this factorisation is initial. Under further assumptions H is a Hopf-algebra. We then consider possible generalizations of this situation.

16:00-17:00 Discussion Panel

A Panel of discussion on the current condition and possible future directions in research topics discussed during the workshop.

Place: IMBM Seminar Room, South Campus Boğaziçi University

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